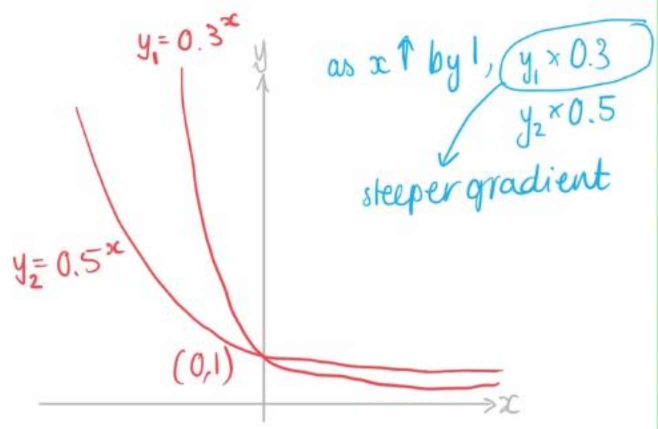
Q1a

(a)



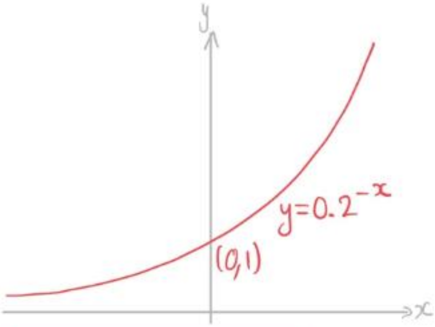
asymptotes: $y=0$ for both curves.

Q1b

$$y = 0.5^{(-1)x} = 0.5^{-x}$$

$$\boxed{y = 0.5^{-x}}$$

Q2a



$y = \dfrac{1}{0.2^x}$

as $x \uparrow$, $y \uparrow$

at $x=0$, $y = 0.2^{-(0)} = 1$

$\boxed{\text{asymptote: } y=0}$

$\boxed{\text{exponential growth}}$

Q2b

$625 = 5^4$

$$625 = 5^4 = 0.2^{-x} = \frac{1}{0.2^x} = \frac{1}{\frac{1^x}{5^x}} = 5^x$$

equate powers

$$\boxed{x=4}$$

Q3a

Let $x = \log_4 128$

$4^x = 128$

$2^2 = 4$ $\quad$ $2^7 = 128$ $\leftarrow$ We have two powers of 2, so rewrite them as powers!

$(2^2)^x = 2^7 = 2^{2x}$

$7 = 2x$

$x = \frac{7}{2}$

$$\boxed{\log_4 128 = \frac{7}{2}}$$

Q3b

$$\frac{3(3) - 5 + 4^4}{4} = \boxed{65}$$

Q4

$$3^{2(x+1)} = 3^{2x+2} = (3^x)^2(3^2) = 9(3^x)^2$$

$$9(3^x)^2 + 3 - 28(3^x) = 0$$

Sub $y = 3^x$

$$9y^2 - 28y + 3 = 0$$

$\times 27$
$+ -28$
$-27, -1$

$$\underbrace{9y^2 - 27y}\ \underbrace{-1y + 3}$$

$$9y(y-3) - (y-3)$$

$$(9y-1)(y-3) = 0$$

$$y = \frac{1}{9}, 3$$

$$3^x = \frac{1}{9} \qquad 3^x = 3$$

Solve by observation, or...

$$3^x = \frac{1}{3^2} \qquad x = 1$$

$$3^x = 3^{-2}$$

$$x = -2$$

... alternatively, solve using logs!

$$x = \log_3\left(\frac{1}{9}\right) = -2$$

$$\boxed{x = 1, -2}$$

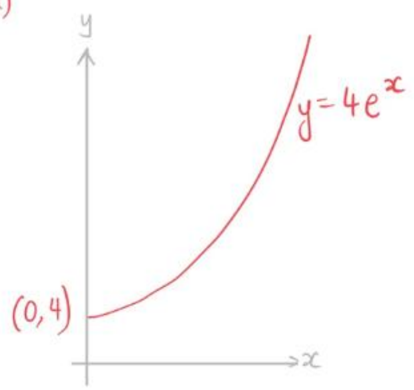
Q5

x			
0 ✗	$\log(0^2)$	math error!	
1 ✓	$\log(1) = 0$		$(\log 1)^2 = 0$
		$\therefore$ LHS = RHS	
10 ✗	$\log(10^2)$ 2	$\neq$	$(\log 10)^2$ 1^2
100 ✓	$\log(100^2)$ 4		$(\log 100)^2$ $2^2 = 4$
		LHS = RHS	

$$\boxed{x = 1,\ 100}$$

Q6a

a)



$$\boxed{\text{asymptote: } y = 0}$$

Q6b

$$\frac{dy}{dx} = 4e^{x}$$

sub $x = 3$

$$\frac{dy}{dx} = 4e^{3} = \boxed{80.3} \quad (3sf)$$

Q6c

$t = 0$ (initial condition)

sub into equation:

$$P = 4\cancel{e^{0}}^{\,1} = \boxed{4}$$

Q7a

$$f(4x) = 5e^{3(4x)} = 5e^{12x}$$

$$\boxed{f(4x) = 5e^{12x}}$$

Q7b

Differenciate wrt x

$$f'(x) = 5(3)e^{3x} = 15e^{3x}$$

substitute

$x \to 5x$

$$f'(5x) = 15e^{3(5x)} = \boxed{15e^{15x}}$$

Q8a

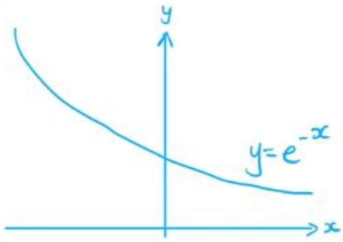
$$\frac{dy}{dx} = \frac{1}{a}(-b)\,e^{-bx} = \boxed{-\frac{b}{a}e^{-bx}}$$

Q8b

e must be raised to a -ve power to show exponential decay

$$-bx < 0$$

$$\boxed{b > 0}$$



Q8c

$$\frac{dy}{dx} = \frac{-b}{a}\cancel{e^{-b(0)}}^{1} = -\frac{b}{a} = 10$$

$$b = -10a$$

sub $b=-10a$ into equation to eliminate b.

$$y = \frac{1}{a}e^{-(-10a)x}$$

$$\boxed{y = \frac{1}{a}e^{10ax}}$$

Q9a

initially $t=0$

$$v = 0.3\cancel{e^{k0}}^{1} = \boxed{0.3\,\text{ms}^{-1}}$$

Q9b

acceleration: gradient v-t graph

$$\frac{dv}{dt} = \boxed{0.3ke^{kt}}$$

Q9c

$t = 12 \quad v = 0.9$

$$0.9 = 0.3e^{k12}$$

$$3 = e^{12k}$$

$$\ln 3 = 12k$$

$$k = \frac{1}{12}\ln 3 = \boxed{0.0916} \quad (3sf)$$

Q9d

- Model suggests the velocity will continue to increase forever so at large values of t there will be some very quick and unrealistic velocities predicted.

- Same applies for acceleration.

Q10

$$(e^x - e^{-x})(e^x - e^{-x}) = 0$$

$$(e^x)^2 - 2e^x e^{-x} + (e^{-x})^2 = 0$$

$$(e^x)^2 - 2(1) + \frac{1}{(e^x)^2} = 0$$

$$(e^x)^4 - 2(e^x)^2 + 1 = 0$$

Let $y = e^{2x}$

$$y^2 - 2y + 1 = 0$$

$$(y-1)(y-1) = 0$$

$$y = 1 = e^{2x}$$

$$\ln 1 = 2x$$

$$x = \frac{1}{2}\cancel{\ln 1}^{0}$$

$$\boxed{x = 0}$$

Q11

$$2(e^x)^{\cancel{3}\,2} - 11(e^x)^{\cancel{2}} + 12\cancel{e^x} = 0$$

$$2(e^x)^2 - 11(e^x) + 12 = 0$$

Let $y = e^x$

$$2y^2 - 11y + 12 = 0$$

$$(y-4)(2y-3) = 0$$

$$y = 4, \frac{3}{2}$$

$$e^x = 4 \qquad e^x = \frac{3}{2}$$

$$x = \ln 4, \quad \ln\frac{3}{2}$$

$$\boxed{x = 1.39, 0.405} \ (3sf)$$